

EXECUTIVE SUMMARY - ENFORCEMENT MATTER
DOCKET NO.: 2006-1969-AIR-E TCEQ ID: RN102382041 CASE NO.: 31720
RESPONDENT NAME: Rental Service Corporation

ORDER TYPE:

<input checked="" type="checkbox"/> 1660 AGREED ORDER	<input type="checkbox"/> FINDINGS AGREED ORDER	<input type="checkbox"/> AMENDED ORDER	<input type="checkbox"/> IMMINENT AND SUBSTANTIAL ENDANGERMENT ORDER
<input type="checkbox"/> SHUTDOWN ORDER	<input type="checkbox"/> FINDINGS DEFAULT ORDER	<input type="checkbox"/> EMERGENCY ORDER	

CASE TYPE:

<input type="checkbox"/> AGRICULTURE	<input checked="" type="checkbox"/> AIR	<input type="checkbox"/> INDUSTRIAL AND HAZARDOUS WASTE	<input type="checkbox"/> MUNICIPAL SOLID WASTE
<input type="checkbox"/> OCCUPATIONAL CERTIFICATION	<input type="checkbox"/> PETROLEUM STORAGE TANKS	<input type="checkbox"/> PUBLIC WATER SUPPLY	<input type="checkbox"/> RADIOACTIVE WASTE
<input type="checkbox"/> MULTI-MEDIA (check all that apply)	<input type="checkbox"/> SEWAGE SLUDGE	<input type="checkbox"/> UNDERGROUND INJECTION CONTROL	<input type="checkbox"/> USED OIL
<input type="checkbox"/> USED OIL FILTER	<input type="checkbox"/> WATER QUALITY		

SITE WHERE VIOLATION(S) OCCURRED: Rental Service Corp Store 670, 6914 Gateway Boulevard East, El Paso, El Paso County

TYPE OF OPERATION: Equipment rental company

SMALL BUSINESS: Yes No

OTHER SIGNIFICANT MATTERS: There are no complaints. There is no record of additional pending enforcement actions regarding this facility location.

INTERESTED PARTIES: No one other than the ED and the Respondent has expressed an interest in this matter.

COMMENTS RECEIVED: The *Texas Register* comment period expired on April 9, 2007. No comments were received.

CONTACTS AND MAILING LIST:

TCEQ Attorney/SEP Coordinator: None

TCEQ Enforcement Coordinator: Ms. Rebecca Johnson, Enforcement Division, Enforcement Section III, MC R-12, (713) 422-8931; Mr. Steven Lopez, Enforcement Division, MC 219, (512) 239-1896

Respondent: Mr. Robert Whitacre, Regulatory Support Manager, Rental Service Corporation, 6929 East Greenway Parkway, Suite 200, Scottsdale, Arizona 85254
Mr. Shyam Srinivas, Environmental Manager, Rental Service Corporation, 6929 East Greenway Parkway, Suite 200, Scottsdale, Arizona 85254

Respondent's Attorney: Not represented by counsel on this enforcement matter

RESPONDENT'S NAME: Rental Service Corporation
DOCKET NO.: 2006-1969-AIR-E

VIOLATION SUMMARY CHART:

VIOLATION INFORMATION	PENALTY CONSIDERATIONS	CORRECTIVE ACTIONS TAKEN/REQUIRED
<p>Type of Investigation: <input type="checkbox"/> Complaint <input checked="" type="checkbox"/> Routine <input type="checkbox"/> Enforcement Follow-up <input type="checkbox"/> Records Review</p> <p>Date of Complaint Relating to this Case: None</p> <p>Date of Investigation Relating to this Case: October 3, 2006</p> <p>Date of NOE Relating to this Case: October 31, 2006 (NOE)</p> <p>Background Facts: This was a routine investigation. One violation was documented.</p> <p>AIR</p> <p>Failed to ensure a minimum of 2.7% oxygenate in gasoline dispensed in El Paso County from October 1st to March 31st. Specifically, a gasoline sample taken during the October 3, 2006 investigation indicated a 0.0% oxygenate in the gasoline being dispensed from the pump at the Site [30 TEX. ADMIN. CODE § 114.100(a) and TEX. HEALTH & SAFETY CODE § 382.085(b)].</p>	<p>Total assessed: \$1,200</p> <p>Total Deferred: \$240 <input checked="" type="checkbox"/> Expedited Settlement <input type="checkbox"/> Financial Inability to Pay</p> <p>SEP Conditional Offset: \$0</p> <p>Total Paid to General Revenue: \$960</p> <p>Site Compliance History Classification: <input type="checkbox"/> High <input checked="" type="checkbox"/> Average <input type="checkbox"/> Poor</p> <p>Person Compliance History Classification: <input type="checkbox"/> High <input checked="" type="checkbox"/> Average <input type="checkbox"/> Poor</p> <p>Major Source: <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No</p> <p>Applicable Penalty Policy: September 2002</p>	<p>Ordering Provisions:</p> <p>The Order will require the Respondent to:</p> <p>a. Within 30 days after the effective date of this Agreed Order, implement measures designed to ensure that gasoline dispensed at the Site from October 1st to March 31st has a minimum of 2.7% oxygenate; and</p> <p>b. Within 45 days after the effective date of this Agreed Order, submit written certification and include detailed supporting documentation including photographs, receipts, and/or other records to demonstrate compliance with Ordering Provision a.</p>



Penalty Calculation Worksheet (PCW)

TCEQ

DATES	Assigned	30-Oct-2006	Screening	13-Nov-2006	EPA Due	
	PCW	17-Nov-2006				

RESPONDENT/FACILITY INFORMATION	
Respondent	Rental Service Corporation
Reg. Ent. Ref. No.	RN102382041
Facility/Site Region	6-El Paso
Major/Minor Source	Minor

CASE INFORMATION			
Enf./Case ID No.	31720	No. of Violations	1
Docket No.	2006-1969-AIR-E	Order Type	1660
Media Program(s)	Air Quality	Enf. Coordinator	Rebecca Johnson
Multi-Media		EC's Team	Enforcement Team 6
Admin. Penalty \$	Limit Minimum \$0	Maximum	\$10,000

Penalty Calculation Section

TOTAL BASE PENALTY (Sum of violation base penalties)	Subtotal 1	\$1,000
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ADJUSTMENTS (+/-) TO SUBTOTAL 1

Subtotals 2-7 are obtained by multiplying the Total Base Penalty (Subtotal 1) by the indicated percentage.

Compliance History	20% Enhancement	Subtotals 2, 3, & 7	\$200
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Notes: Penalty enhancement due to one agreed order containing a denial of liability.

Culpability	No	0% Enhancement	Subtotal 4	\$0
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Notes: Respondent does not meet the culpability criteria.

Good Faith Effort to Comply	0% Reduction	Subtotal 5	\$0
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Before NOV NOV to EDRP/Settlement Offer

Extraordinary		
Ordinary		
N/A	X	(mark with x)

Notes: The Respondent does not meet the good faith criteria.

Total EB Amounts	\$271	0% Enhancement*	Subtotal 6	\$0
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Approx. Cost of Compliance \$850 *Capped at the Total EB \$ Amount

SUM OF SUBTOTALS 1-7	Final Subtotal	\$1,200
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OTHER FACTORS AS JUSTICE MAY REQUIRE	Adjustment	\$0
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Reduces or enhances the Final Subtotal by the indicated percentage. (Enter number only; e.g. -30 for -30%.)

Notes:

Final Penalty Amount	\$1,200
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STATUTORY LIMIT ADJUSTMENT	Final Assessed Penalty	\$1,200
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DEFERRAL	20% Reduction	Adjustment	-\$240
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Reduces the Final Assessed Penalty by the indicated percentage. (Enter number only; e.g. 20 for 20% reduction.)

Notes:

Deferral offered for expedited settlement.

PAYABLE PENALTY	\$960
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Screening Date 13-Nov-2006

Docket No. 2006-1969-AIR-E

PCW

Respondent Rental Service Corporation

Policy Revision 2 (September 2002)

Case ID No. 31720

PCW Revision November 17, 2006

Reg. Ent. Reference No. RN102382041

Media [Statute] Air Quality

Enf. Coordinator Rebecca Johnson

Compliance History Worksheet

>> Compliance History Site Enhancement (Subtotal 2)

Component	Number of...	Enter Number Here	Adjust.
NOVs	Written NOVs with same or similar violations as those in the current enforcement action (number of NOVs meeting criteria)	0	0%
	Other written NOVs	0	0%
Orders	Any agreed final enforcement orders containing a denial of liability (number of orders meeting criteria)	1	20%
	Any adjudicated final enforcement orders, agreed final enforcement orders without a denial of liability, or default orders of this state or the federal government, or any final prohibitory emergency orders issued by the commission	0	0%
Judgments and Consent Decrees	Any non-adjudicated final court judgments or consent decrees containing a denial of liability of this state or the federal government (number of judgments or consent decrees meeting criteria)	0	0%
	Any adjudicated final court judgments and default judgments, or non-adjudicated final court judgments or consent decrees without a denial of liability, of this state or the federal government	0	0%
Convictions	Any criminal convictions of this state or the federal government (number of counts)	0	0%
Emissions	Chronic excessive emissions events (number of events)	0	0%
Audits	Letters notifying the executive director of an intended audit conducted under the Texas Environmental, Health, and Safety Audit Privilege Act, 74th Legislature, 1995 (number of audits for which notices were submitted)	0	0%
	Disclosures of violations under the Texas Environmental, Health, and Safety Audit Privilege Act, 74th Legislature, 1995 (number of audits for which violations were disclosed)	0	0%
<i>Please Enter Yes or No</i>			
Other	Environmental management systems in place for one year or more	No	0%
	Voluntary on-site compliance assessments conducted by the executive director under a special assistance program	No	0%
	Participation in a voluntary pollution reduction program	No	0%
	Early compliance with, or offer of a product that meets future state or federal government environmental requirements	No	0%

Adjustment Percentage (Subtotal 2) 20%

>> Repeat Violator (Subtotal 3)

No

Adjustment Percentage (Subtotal 3) 0%

>> Compliance History Person Classification (Subtotal 7)

Average Performer

Adjustment Percentage (Subtotal 7) 0%

>> Compliance History Summary

Compliance History Notes

Penalty enhancement due to one agreed order containing a denial of liability.

Total Adjustment Percentage (Subtotals 2, 3, & 7) 20%

Screening Date 13-Nov-2006

Docket No. 2006-1969-AIR-E

PCW

Respondent Rental Service Corporation

Policy Revision 2 (September 2002)

Case ID No. 31720

PCW Revision November 17, 2006

Reg. Ent. Reference No. RN102382041

Media [Statute] Air Quality

Enf. Coordinator Rebecca Johnson

Violation Number 1

Rule Cite(s) 30 Tex. Admin. Code § 114.100(a) and Tex. Health & Safety Code § 382.085(b)

Violation Description Failed to ensure a minimum of 2.7% oxygenate in gasoline dispensed in El Paso County from October 1st to March 31st. Specifically, a gasoline sample taken during the October 3, 2006 investigation indicated a 0.0% oxygenate in the gasoline being dispensed from the pump at the site.

Base Penalty \$10,000

>> Environmental, Property and Human Health Matrix

Release	Harm			Percent
	Major	Moderate	Minor	
Actual			X	10%
Potential				

>> Programmatic Matrix

Falsification	Major	Moderate	Minor	Percent
				0%

Matrix Notes Human health or the environment has been exposed to insignificant amounts of pollutants which did not exceed levels that are protective of human health or environmental receptors as a result of the violation.

Adjustment \$9,000

\$1,000

Violator 250 10/3/2006 3/31/2006

Number of Violation Events 1 Number of violation days 41

daily	
monthly	
quarterly	
semiannual	
annual	
single event	X

mark only one with an x

Violation Base Penalty \$1,000

One single event is recommended based on the non-compliant sample that was taken during the October 3, 2006 investigation.

Economic Benefit (EB) for this violation

Statutory Limit Test

Estimated EB Amount \$271

Violation Final Penalty Total \$1,200

This violation Final Assessed Penalty (adjusted for limits) \$1,200

Economic Benefit Worksheet

Respondent: Rental Service Corporation
Case ID No.: 31720
Reg. Ent. Reference No.: RN102382041
Media: Air Quality
Violation No.: 1

Percent Interest	Years of Depreciation
5.0	15

Item Description	Item Cost	Date Required	Final Date	Yrs	Interest Saved	Onetime Costs	EB Amount
No commas or \$							

Delayed Costs

Equipment				0.0	\$0	\$0	\$0
Buildings				0.0	\$0	\$0	\$0
Other (as needed)	\$600	3-Oct-2006	31-Mar-2007	0.5	\$1	\$20	\$21
Engineering/construction				0.0	\$0	\$0	\$0
Land				0.0	\$0	n/a	\$0
Record Keeping System				0.0	\$0	n/a	\$0
Training/Sampling				0.0	\$0	n/a	\$0
Remediation/Disposal				0.0	\$0	n/a	\$0
Permit Costs				0.0	\$0	n/a	\$0
Other (as needed)				0.0	\$0	n/a	\$0

Notes for DELAYED costs: Estimated cost to implement measures designed to ensure that compliant gasoline is dispensed during the control period. Date required is based on the investigation date. Final date is based on the projected compliance date.

Avoided Costs

ANNUALIZE [1] avoided costs before entering item (except for one-time avoided costs)

Disposal				0.0	\$0	\$0	\$0
Personnel				0.0	\$0	\$0	\$0
Inspection/Reporting/Sampling				0.0	\$0	\$0	\$0
Supplies/equipment				0.0	\$0	\$0	\$0
Financial Assurance [2]				0.0	\$0	\$0	\$0
ONE-TIME avoided costs [3]				0.0	\$0	\$0	\$0
Other (as needed)	\$250	3-Oct-2006	31-Mar-2006	0.0	\$0	\$250	\$250

Notes for AVOIDED costs: The avoided cost is based on dispensing and utilizing gasoline without the minimum oxygen content. Date required is the investigation date. Final date is based on the estimated date of compliance.

Approx. Cost of Compliance	\$850	TOTAL	\$271
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N/A

H. Voluntary on-site compliance assessment dates.

N/A

I. Participation in a voluntary pollution reduction program.

N/A

J. Early compliance.

N/A

Sites Outside of Texas

N/A

09/08/2018

09/08/2018

09/08/2018

09/08/2018

09/08/2018

09/08/2018

09/08/2018

09/08/2018

TEXAS COMMISSION ON ENVIRONMENTAL QUALITY



IN THE MATTER OF AN
ENFORCEMENT ACTION
CONCERNING
RENTAL SERVICE CORPORATION
RN102382041

§
§
§
§
§

BEFORE THE
TEXAS COMMISSION ON
ENVIRONMENTAL QUALITY

AGREED ORDER
DOCKET NO. 2006-1969-AIR-E

I. JURISDICTION AND STIPULATIONS

At its _____ agenda, the Texas Commission on Environmental Quality ("the Commission" or "TCEQ") considered this agreement of the parties, resolving an enforcement action regarding Rental Service Corporation ("RSC") under the authority of TEX. HEALTH & SAFETY CODE ch. 382 and TEX. WATER CODE ch. 7. The Executive Director of the TCEQ, through the Enforcement Division, and RSC appear before the Commission and together stipulate that:

1. RSC owns and operates an equipment rental company at 6914 Gateway Boulevard East in El Paso, El Paso County, Texas (the "Site").
2. The Site consists of one or more sources as defined in TEX. HEALTH & SAFETY CODE § 382.003(12).
3. The Commission and RSC agree that the Commission has jurisdiction to enter this Agreed Order, and that RSC is subject to the Commission's jurisdiction.
4. RSC received notice of the violations alleged in Section II ("Allegations") on or about November 5, 2006.
5. The occurrence of any violation is in dispute and the entry of this Agreed Order shall not constitute an admission by RSC of any violation alleged in Section II ("Allegations"), nor of any statute or rule.
6. An administrative penalty in the amount of One Thousand Two Hundred Dollars (\$1,200) is assessed by the Commission in settlement of the violations alleged in Section II ("Allegations"). RSC has paid Nine Hundred Sixty Dollars (\$960) of the administrative penalty and Two Hundred

Forty Dollars (\$240) is deferred contingent upon RSC's timely and satisfactory compliance with all the terms of this Agreed Order. The deferred amount will be waived upon full compliance with the terms of this Agreed Order. If RSC fails to timely and satisfactorily comply with all requirements of this Agreed Order, the Executive Director may require RSC to pay all or part of the deferred penalty.

7. Any notice and procedures, which might otherwise be authorized or required in this action, are waived in the interest of a more timely resolution of the matter.
8. The Executive Director of the TCEQ and RSC have agreed on a settlement of the matters alleged in this enforcement action, subject to the approval of the Commission.
9. The Executive Director may, without further notice or hearing, refer this matter to the Office of the Attorney General of the State of Texas ("OAG") for further enforcement proceedings if the Executive Director determines that RSC has not complied with one or more of the terms or conditions in this Agreed Order.
10. This Agreed Order shall terminate five years from its effective date or upon compliance with all the terms and conditions set forth in this Agreed Order, whichever is later.
11. The provisions of this Agreed Order are deemed severable and, if a court of competent jurisdiction or other appropriate authority deems any provision of this Agreed Order unenforceable, the remaining provisions shall be valid and enforceable.

II. ALLEGATIONS

As owner and operator of the Site, RSC is alleged to have failed to ensure a minimum of 2.7% oxygenate in gasoline dispensed in El Paso County from October 1st to March 31st, in violation of 30 TEX. ADMIN. CODE § 114.100(a) and TEX. HEALTH & SAFETY CODE § 382.085(b), as documented during an investigation conducted on October 3, 2006. Specifically, a gasoline sample taken during the October 3, 2006 investigation indicated a 0.0% oxygenate in the gasoline being dispensed from the pump at the Site.

III. DENIALS

RSC generally denies each allegation in Section II ("Allegations").

IV. ORDERING PROVISIONS

1. It is, therefore, ordered by the TCEQ that RSC pay an administrative penalty as set forth in Section I, Paragraph 6 above. The payment of this administrative penalty and RSC's compliance with all the terms and conditions set forth in this Agreed Order resolve only the allegations in Section II. The Commission shall not be constrained in any manner from requiring corrective action or penalties for violations, which are not raised here. Administrative penalty payments shall be made payable to "TCEQ" and shall be sent with the notation "Re: Rental Service Corporation, Docket No. 2006-1969-AIR-E" to:

Financial Administration Division, Revenues Section
Attention: Cashier's Office, MC 214
Texas Commission on Environmental Quality
P.O. Box 13088
Austin, Texas 78711-3088

2. It is further ordered that RSC shall undertake the following technical requirements:
 - a. Within 30 days after the effective date of this Agreed Order, implement measures designed to ensure that gasoline dispensed at the Site from October 1st to March 31st has a minimum of 2.7% oxygenate; and
 - b. Within 45 days after the effective date of this Agreed Order, submit written certification as described below, and include detailed supporting documentation including photographs, receipts, and/or other records to demonstrate compliance with Ordering Provision No. 2.a. The certification shall be notarized by a State of Texas Notary Public and include the following certification language:

"I certify under penalty of law that I have personally examined and am familiar with the information submitted and all attached documents, and that based on my inquiry of those individuals immediately responsible for obtaining the information, I believe that the submitted information is true, accurate and complete. I am aware that there are significant penalties for submitting false information, including the possibility of fine and imprisonment for knowing violations."

The certification shall be submitted to:

Order Compliance Team
Enforcement Division, MC 149A
Texas Commission on Environmental Quality
P.O. Box 13087
Austin, Texas 78711-3087

with a copy to:

Air Quality Section, Manager
El Paso Regional Office
Texas Commission on Environmental Quality
401 East Franklin Avenue, Suite 560
El Paso, Texas 79901-1206

3. The provisions of this Agreed Order shall apply to and be binding upon RSC. RSC is ordered to give notice of the Agreed Order to personnel who maintain day-to-day control over the operations referenced in this Agreed Order.
4. If RSC fails to comply with any of the Ordering Provisions in this Agreed Order within the prescribed schedules, and that failure is caused solely by an act of God, war, strike, riot, or other catastrophe, RSC's failure to comply is not a violation of this Agreed Order. RSC shall have the burden of establishing to the Executive Director's satisfaction that such an event has occurred. RSC shall notify the Executive Director within seven days after RSC becomes aware of a delaying event and shall take all reasonable measures to mitigate and minimize any delay.
5. The Executive Director may grant an extension of any deadline in this Agreed Order or in any plan, report, or other document submitted pursuant to this Agreed Order, upon a written and substantiated showing of good cause. All requests for extensions by RSC shall be made in writing to the Executive Director. Extensions are not effective until RSC receives written approval from the Executive Director. The determination of what constitutes good cause rests solely with the Executive Director.
6. This Agreed Order, issued by the Commission, shall not be admissible against RSC in a civil proceeding, unless the proceeding is brought by the OAG to: (1) enforce the terms of this Agreed Order; or (2) pursue violations of a statute within the Commission's jurisdiction, or of a rule adopted or an order or permit issued by the Commission under such a statute.
7. This agreement may be executed in multiple counterparts, which together shall constitute a single original instrument. Any executed signature page to this Agreement may be transmitted by facsimile transmission to the other parties, which shall constitute an original signature for all purposes.
8. Under 30 TEX. ADMIN. CODE § 70.10(b), the effective date is the date of hand-delivery of the Order to RSC, or three days after the date on which the Commission mails notice of the Order to RSC, whichever is earlier. The Chief Clerk shall provide a copy of this Agreed Order to each of the parties.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that this is crucial for ensuring transparency and accountability in the organization's operations.

2. The second part of the document outlines the various methods and tools used to collect and analyze data. It highlights the need for consistent and reliable data collection processes to ensure the validity of the findings.

3. The third part of the document describes the results of the data analysis. It shows that there is a significant correlation between the variables studied, which supports the hypothesis that was tested.

4. The fourth part of the document discusses the implications of the findings. It suggests that the results can be used to inform decision-making and to develop strategies to improve the organization's performance.

5. The fifth part of the document concludes the study and provides a summary of the key findings. It also identifies some limitations of the study and suggests areas for future research.

6. The final part of the document provides a list of references and a bibliography. It includes all the sources that were consulted during the research process.

Rental Service Corporation
DOCKET NO. 2006-1969-AIR-E
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SIGNATURE PAGE

TEXAS COMMISSION ON ENVIRONMENTAL QUALITY

For the Commission



For the Executive Director

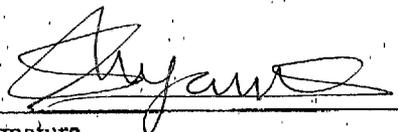
5/7/07
Date

I, the undersigned, have read and understand the attached Agreed Order. I am authorized to agree to the attached Agreed Order on behalf of the entity, if any, indicated below my signature, and I do agree to the terms and conditions specified therein. I further acknowledge that the TCEQ, in accepting payment for the penalty amount, is materially relying on such representation.

I also understand that my failure to comply with the Ordering Provisions, if any, in this order and/or my failure to timely pay the penalty amount, may result in:

- A negative impact on my compliance history;
- Greater scrutiny of any permit applications submitted by me;
- Referral of this case to the Attorney General's Office for contempt, injunctive relief, additional penalties, and/or attorney fees, or to a collection agency;
- Increased penalties in any future enforcement actions against me;
- Automatic referral to the Attorney General's Office of any future enforcement actions against me; and
- TCEQ seeking other relief as authorized by law.

In addition, any falsification of any compliance documents may result in criminal prosecution.



Signature

02/07/07
Date

SHYAM SRINIVAS
Name (Printed or typed)
Authorized Representative of
Rental Service Corporation

ENVIRONMENTAL MANAGER
Title

Instructions: Send the original, signed Agreed Order with penalty payment to the Financial Administration Division, Revenues Section at the address in Section IV, Paragraph 1 of this Agreed Order.

Mathematical Induction



Let $P(n)$ be a statement involving the natural number n . To prove that $P(n)$ is true for all natural numbers n , we use the principle of mathematical induction. The first step is to prove that $P(1)$ is true. This is the base case. If $P(1)$ is true, then we assume that $P(k)$ is true for some natural number k . This is the inductive hypothesis. We then prove that $P(k+1)$ is true. This is the inductive step. If we can prove that $P(k+1)$ is true whenever $P(k)$ is true, then by the principle of mathematical induction, $P(n)$ is true for all natural numbers n .

Example: Prove that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ for all natural numbers n .
 Let $P(n)$ be the statement $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.
 Base Case: For $n=1$, $1 = \frac{1(1+1)}{2} = 1$. So $P(1)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$. We need to prove that $P(k+1)$ is true, i.e., $1 + 2 + 3 + \dots + k + (k+1) = \frac{(k+1)(k+1+1)}{2}$.
 Starting from the inductive hypothesis, we add $(k+1)$ to both sides:
 $1 + 2 + 3 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$
 $= \frac{k(k+1) + 2(k+1)}{2}$
 $= \frac{(k+1)(k+2)}{2}$
 So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers n .

Another example: Prove that $2^n > n$ for all natural numbers n .
 Let $P(n)$ be the statement $2^n > n$.
 Base Case: For $n=1$, $2^1 = 2 > 1$. So $P(1)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $2^k > k$. We need to prove that $P(k+1)$ is true, i.e., $2^{k+1} > k+1$.
 Starting from the inductive hypothesis, we multiply both sides by 2:
 $2 \cdot 2^k > 2 \cdot k$
 $2^{k+1} > 2k$
 Since $2k > k+1$ for all natural numbers k , we have $2^{k+1} > 2k > k+1$. So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers n .

Example: Prove that $n^2 > n$ for all natural numbers $n > 1$.
 Let $P(n)$ be the statement $n^2 > n$.
 Base Case: For $n=2$, $2^2 = 4 > 2$. So $P(2)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $k^2 > k$. We need to prove that $P(k+1)$ is true, i.e., $(k+1)^2 > k+1$.
 Starting from the inductive hypothesis, we add $2k+1$ to both sides:
 $k^2 + 2k + 1 > k + 2k + 1$
 $(k+1)^2 > 3k + 1$
 Since $3k + 1 > k + 1$ for all natural numbers k , we have $(k+1)^2 > 3k + 1 > k + 1$. So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers $n > 1$.

Example: Prove that $3^n > n^3$ for all natural numbers $n > 3$.
 Let $P(n)$ be the statement $3^n > n^3$.
 Base Case: For $n=4$, $3^4 = 81 > 4^3 = 64$. So $P(4)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $3^k > k^3$. We need to prove that $P(k+1)$ is true, i.e., $3^{k+1} > (k+1)^3$.
 Starting from the inductive hypothesis, we multiply both sides by 3:
 $3 \cdot 3^k > 3 \cdot k^3$
 $3^{k+1} > 3k^3$
 We need to show that $3k^3 > (k+1)^3$.
 $3k^3 > k^3 + 3k^2 + 3k + 1$
 $2k^3 > 3k^2 + 3k + 1$
 For $k > 3$, $2k^3 > 3k^2 + 3k + 1$. So $3^{k+1} > 3k^3 > (k+1)^3$. So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers $n > 3$.

Example: Prove that $2^n > n^2$ for all natural numbers $n > 4$.
 Let $P(n)$ be the statement $2^n > n^2$.
 Base Case: For $n=5$, $2^5 = 32 > 5^2 = 25$. So $P(5)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $2^k > k^2$. We need to prove that $P(k+1)$ is true, i.e., $2^{k+1} > (k+1)^2$.
 Starting from the inductive hypothesis, we multiply both sides by 2:
 $2 \cdot 2^k > 2 \cdot k^2$
 $2^{k+1} > 2k^2$
 We need to show that $2k^2 > (k+1)^2$.
 $2k^2 > k^2 + 2k + 1$
 $k^2 > 2k + 1$
 For $k > 4$, $k^2 > 2k + 1$. So $2^{k+1} > 2k^2 > (k+1)^2$. So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers $n > 4$.

Example: Prove that $2^n > n^3$ for all natural numbers $n > 10$.
 Let $P(n)$ be the statement $2^n > n^3$.
 Base Case: For $n=11$, $2^{11} = 2048 > 11^3 = 1331$. So $P(11)$ is true.
 Inductive Step: Assume $P(k)$ is true, i.e., $2^k > k^3$. We need to prove that $P(k+1)$ is true, i.e., $2^{k+1} > (k+1)^3$.
 Starting from the inductive hypothesis, we multiply both sides by 2:
 $2 \cdot 2^k > 2 \cdot k^3$
 $2^{k+1} > 2k^3$
 We need to show that $2k^3 > (k+1)^3$.
 $2k^3 > k^3 + 3k^2 + 3k + 1$
 $k^3 > 3k^2 + 3k + 1$
 For $k > 10$, $k^3 > 3k^2 + 3k + 1$. So $2^{k+1} > 2k^3 > (k+1)^3$. So $P(k+1)$ is true. By the principle of mathematical induction, $P(n)$ is true for all natural numbers $n > 10$.